Book Review

Roman G. Strongin, Yaroslav D. Sergeyev, Global Optimization with Non-Convex Constraints: Sequential and Parallel Algorithms, Nonconvex Optimization and Its Applications, Vol. 45, Kluwer Academic Publishers, Dordrecht, 2000, 728 pp.

Very often in real-life applications both the objective function and constraints can be multiextremal, non-differentiable, partially defined, and hard to be evaluated. Thus, such problems can not be solved by traditional optimization techniques making strong suppositions about the problem (differentiability, convexity, etc.). A possible attempt to describe a wide class of such problems is based on a quite natural assumption that any limited change in the parameters of the optimized model leads to some limited changes in the model's performance. This assumption is justified by the fact that in technical systems the energy of change is always bounded and can be expressed mathematically in the form of the Lipschitz condition. The book under review is the first comprehensive presentation of an original approach developed by the authors during the past thirty years for solving Lipschitz global optimization problems (previous presentations were not so complete and very often are available only in Russian).

The approach combines a number of innovative and powerful proposals leading to new optimization tools. The following are worthy of a special notice: fractal approach for reduction of the problem dimension by using Peano-Hilbert space-filling curves, index scheme for treating constraints, non-redundant parallel computations, and local tuning on behavior of the objective function and constraints over different subregions of the search domain for accelerating the search.

The contents of the book is split in three parts:

Part 1 considers the core one-dimensional case dealing with optimization of a "black-box" Lipschitz function over a closed interval. This case is studied in depth because the rest of the book shows that the multi-dimensional problem with multiextremal constraints can be reduced to this key case by tools developed in Parts 2 and 3. Two approaches to construction of numerical algorithms, information and geometric, are presented. The first one shows that global optimization algorithms can be obtained as statistical decision procedures. The second approach considers numerical methods as bounding procedures. A detailed convergence theory including some new concepts and estimations (trial densities, topologies of the minimizing sequences, nearly monotonous convergence, etc.) is presented for all

the methods. Special attention is paid to an adaptive estimation of the unknown parameters (the concept of the local tuning is first introduced in this Part).

Part 2 discusses some new approaches for an efficient generalization of the one-dimensional techniques developed in Part 1 for the constrained and multicriteria cases and for implementing these techniques on multiprocessor systems. A specially designed new index scheme is used to reduce the constrained problem to the unconstrained one. This scheme is applicable also in the case when the objective function and/or constraints are not everywhere defined in the search domain. It takes separate account of each constraint and does not use any penalty coefficient or additional variables. The special index scheme is suggested for reducing multicriteria (constrained) problems to some scalar unconstrained ones that are solvable by the new algorithm uniformly approximating the set of weakly effective solutions. The new concept of non-redundant parallelism is introduced to construct efficient global search algorithms for multiprocessor systems. In this context, detailed results concerning convergence conditions and efficiency estimates of the proposed parallel global optimization methods are both of theoretical and practical interest.

Part 3 deals with the full scale problems, i.e., multiextremal, constrained or multicriteria, in many dimensions, and with the possibility to be solved on multiprocessor systems. The techniques suggested are generalizations of the univariate methods from Parts 1 and 2, obtained by employing space-filling Peano-Hilbert curves to reduce dimension of the problem. It introduces the new concept of multiple Peano scannings designed for better translation of multidimensional space metric properties into one-dimensional scales. The important feature of this part is that it contains not only a comprehensive presentation of the theory and numerical algorithms but also the software needed for practical implementations of the techniques based on Peano-Hilbert space-filling curves. Part 3 introduces also parallel methods using the curves and gives a discussion on optimal employment of the available number of processors during global optimization in the case of asynchronous global optimization.

In conclusion, this is a valuable book for both graduate students and researchers in fields such as global optimization, parallel computations, decision making, and related areas. It is well organized and self contained. The detailed index, bibliography, and lists of algorithms, tables, and figures enhance its usefulness. Readers will find this book to be an excellent source of new ideas and future successful developments.

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